A NOVEL APPROACH FOR THE ANALYSIS OF A THYRISTOR-CONTROLLED INDUCTION MOTOR

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Abstract: *In this paper, analytical expressions to obtain directly the extinction angle for the two modes of operation, mode (2/3) and mode (0/2), for a thyristor-controlled induction motor are presented. The extinction angle values obtained analytically are used to obtain performance characteristics of the thyristor-controlled induction motor, and the results were compared with those obtained when numerical methods are used to obtain the extinction angle, and the two sets of results were found to be in very close agreement.*

Keywords: *Thyristor-controlled induction motor, AC voltage controller, Extinction angle.*

1. Introduction

There are various methods for the speed control of induction motors [1, 2]. Thyristor control (phase control), in which thyristors are used to control the motor directly from an AC supply, is one of these methods in which the terminal voltage of the motor is controlled by delaying the instant of application of the supply voltage using AC voltage controller [1-4].

An important parameter required in the analysis of phase-controlled induction motors, using thyristors, is the extinction angle which is usually obtained using numerical techniques to solve a transcendental equation relating the triggering angle of the switches of the AC voltage controller, the phase angle of the motor at certain speed and the extinction angle.

In this paper, analytical expressions of the extinction angle as functions of the triggering angle and the motor phase angle are presented for the two modes of operation, mode (2/3) and mode (0/2).

The fundamental voltage, current and the corresponding torque characteristics as functions of the triggering angle for various motor speeds are obtained using the extinction angle values obtained analytically, and compared with the results obtained using its values obtained numerically.

2. Method of Analysis

In thyristor-controlled induction motor, a 3-phase supply is applied to the motor via a 3-phase AC voltage controller as shown in Fig. 1. The terminal voltage of the motor will be the output voltage of this voltage controller which is nonsinusoidal.

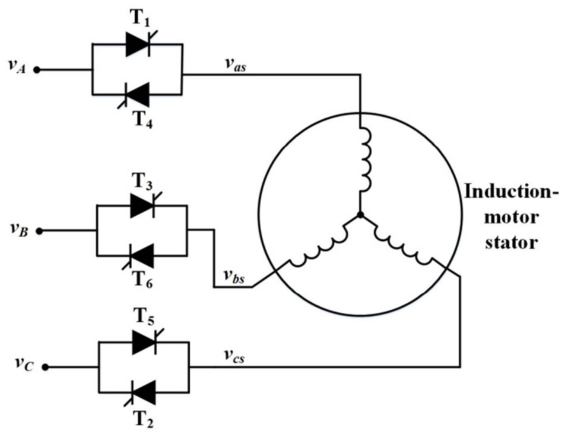

Fig. 1. Phase-controlled induction-motor drive.

The steady-state per-phase equivalent circuit of the three-phase induction motor, at fundamental frequency, is shown in Fig. 2.

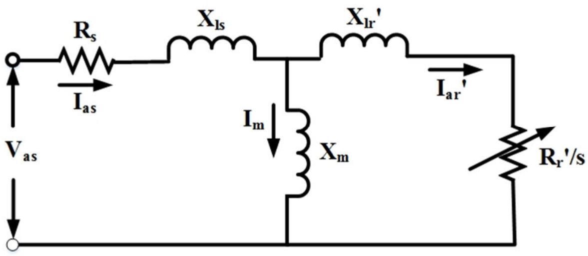

Fig. 2. Steady-state per-phase equivalent circuit of the three-phase induction motor at fundamental frequency.

At certain speed, the induction motor can be dealt with as a series R-L load whose equivalent resistance, R_{in}, and equivalent reactance, X_{in}, can be obtained from the following equations, Fig. 2:

$$R_{in} = R_s + \frac{X_m^2}{\left(\frac{R_r'}{s}\right)^2 + \left(X_m + X_{lr}'\right)^2} \cdot \frac{R_r'}{s} \tag{1}$$

$$X_{in} = X_{ls} + \frac{\left(\frac{R_r'}{s}\right)^2 + X_{lr}'\left(X_m + X_{lr}'\right)}{\left(\frac{R_r'}{s}\right)^2 + \left(X_m + X_{lr}'\right)^2} \cdot X_m \tag{2}$$

where X_m, X_{ls} and X_{lr}' are, respectively, the magnetizing reactance, the stator leakage reactance and the rotor leakage reactance of the motor referred to the stator.

The motor phase angle, ϕ, can be obtained from:

$$\phi = \tan^{-1}\frac{X_{in}}{R_{in}} \tag{3}$$

If at a certain speed, i.e. certain phase angle, the triggering angle, α, is less than the motor phase angle, ϕ, the voltage applied to the motor will be the supply sinusoidal voltage [1, 2, 5-8]. On the other hand, if the triggering angle is higher than the motor phase angle, there will be two modes of operation namely Mode (2/3) and Mode (0/2). The operating mode in this case depends on the value of the triggering angle of the switching elements w.r.t the values of the motor phase angle and the corresponding extinction angle, β [5-10].

2.1. Mode (2/3) $\left(\phi < \alpha < \beta + \frac{\pi}{3}\right)$

In this mode, either two or three thyristors, Fig. 1, are conducting, i.e. two or three lines of the motor are connected to the supply. Typical waveforms of the motor phase-a voltage and current at a certain speed and triggering angle are shown in Fig. 3 [6-11].

The motor phase-a voltage, v_{as}, in this mode, can be expressed mathematically by, Fig. 3 [7, 10]:

$$\begin{aligned}
v_{as} &= V_m \sin(\omega t) && \alpha \le \omega t \le \beta + \frac{\pi}{3} \\
&= \frac{v_{AB}}{2} = \frac{\sqrt{3}}{2} V_m \sin\left(\omega t + \frac{\pi}{6}\right) && \beta + \frac{\pi}{3} \le \omega t \le \alpha + \frac{\pi}{3} \\
&= V_m \sin(\omega t) && \alpha + \frac{\pi}{3} \le \omega t \le \beta + \frac{2\pi}{3} \\
&= \frac{v_{AC}}{2} = \frac{\sqrt{3}}{2} V_m \sin\left(\omega t - \frac{\pi}{6}\right) && \beta + \frac{2\pi}{3} \le \omega t \le \alpha + \frac{2\pi}{3} \\
&= V_m \sin(\omega t) && \alpha + \frac{2\pi}{3} \le \omega t \le \beta + \pi \\
&= 0 && \beta + \pi \le \omega t \le \alpha + \pi \\
&= V_m \sin(\omega t) && \alpha + \pi \le \omega t \le \beta + \frac{4\pi}{3} \\
&= \frac{v_{AB}}{2} = \frac{\sqrt{3}}{2} V_m \sin\left(\omega t + \frac{\pi}{6}\right) && \beta + \frac{4\pi}{3} \le \omega t \le \alpha + \frac{4\pi}{3} \\
&= V_m \sin(\omega t) && \alpha + \frac{4\pi}{3} \le \omega t \le \beta + \frac{5\pi}{3} \\
&= \frac{v_{AC}}{2} = \frac{\sqrt{3}}{2} V_m \sin\left(\omega t - \frac{\pi}{6}\right) && \beta + \frac{5\pi}{3} \le \omega t \le \alpha + \frac{5\pi}{3} \\
&= V_m \sin(\omega t) && \alpha + \frac{5\pi}{3} \le \omega t \le \beta + 2\pi \\
&= 0 && \beta + 2\pi \le \omega t \le \alpha + 2\pi
\end{aligned} \tag{4}$$

where V_m is the maximum value of the sinusoidal supply voltage.

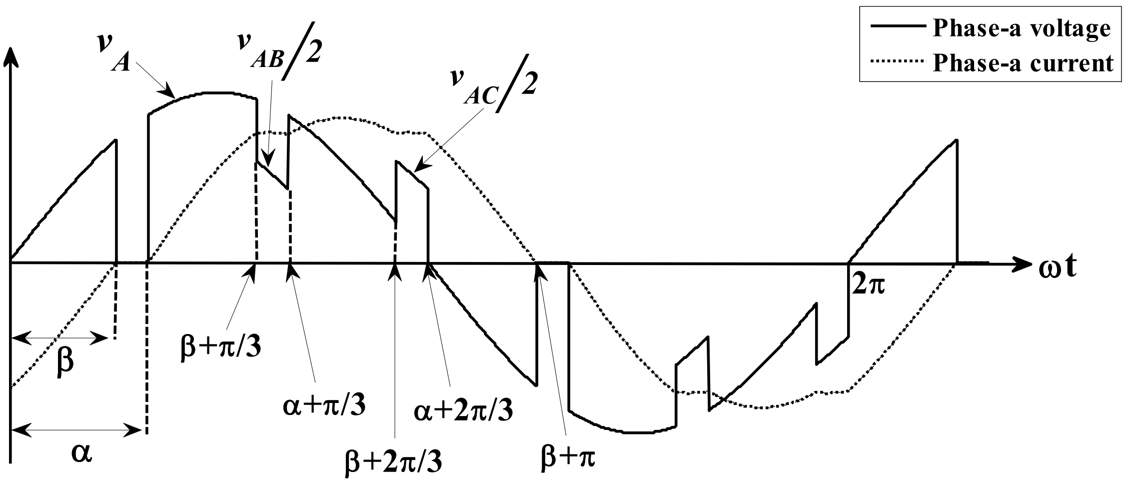

Fig. 3. Induction motor phase-a voltage and current waveforms, $\phi = 45^\circ$, $\alpha = 60^\circ$ (Mode 2/3).

2.1.1 Analytical Determination of the Extinction Angle in Mode (2/3)

In this mode of operation, the extinction angle can be obtained by solving the following transcendental equation, eqn. (5), using any numerical technique [5-11].

$$\sin(\beta-\phi)+K_6\sin(\alpha-\phi)e^{-(\beta-\alpha)\cot\phi}=0 \qquad (5)$$

where:

$$K_6=\frac{2e^{-\pi\cot\phi}+e^{-\frac{2\pi}{3}\cot\varphi}-e^{-\frac{\pi}{3}\cot\phi}}{2-e^{-\frac{2\pi}{3}\cot\varphi}+e^{-\frac{\pi}{3}\cot\phi}} \qquad (6)$$

To obtain an explicit analytical expression for the extinction angle, β, its values from the numerical solution of eqn. (5) are used to obtain the desired analytical expression as a function of the triggering angle and the motor phase angle. The most suitable analytical expression obtained will be as follows [9, 12]:

$$\beta=\phi-K_6(\alpha-\phi)u(\alpha-\phi)+\frac{(\alpha_c-\phi)(1+K_6)-\frac{\pi}{3}}{(\alpha_c-\phi)^3}(\alpha-\phi)^3u(\alpha-\phi) \qquad (7)$$

where:

$$u(\alpha-\phi)=0, \quad \alpha\le\phi$$
$$=1, \quad \alpha>\phi \qquad (8)$$

and α_c is the critical triggering angle, and its expression will be derived in subsection (2.3).
Thus, in this mode, in which $\phi<\alpha<\beta+\pi/3$, the derived explicit analytical expression of the extinction angle will be:

$$\beta=\phi-K_6(\alpha-\phi)+\frac{(1+K_6)-\frac{\pi}{3}}{(\alpha_c-\phi)^2}(\alpha-\phi)^3 \qquad (9)$$

2.1.2 Derivation of Analytical Expressions of the Voltage in Mode (2/3)

Referring to Fig. 3, The Fourier coefficients, a_1 and b_1, of the fundamental component of the motor phase-a voltage can be obtained from the following equations [11]:

$$a_1=\frac{V_m}{\pi}\left[\left(\frac{3}{2}\beta-\frac{3}{2}\alpha+\pi\right)\sin\alpha-\frac{3}{4}\cos(2\beta-\alpha)+\frac{3}{4}\cos\alpha\right] \qquad (10)$$

$$b_1 = \frac{V_m}{\pi}\left[\left(\frac{3}{2}\beta - \frac{3}{2}\alpha + \pi\right)\cos\alpha - \frac{3}{4}\sin\left(2\beta - \alpha\right) + \frac{3}{4}\sin\alpha\right] \tag{11}$$

The RMS value of the fundamental component of the motor phase-a voltage, V_{as1}, can be obtained from:

$$V_{as1} = \sqrt{\frac{a_1^2 + b_1^2}{2}} \tag{12}$$

For the nth harmonic component of the motor phase voltage, the Fourier coefficients are as follows [11]:

$$a_n = \frac{V_m}{2\pi}\left[\cos\frac{n\pi}{3} - \cos\frac{2n\pi}{3} + 2\right]\left[\frac{\cos\alpha - \cos\left(\left(n+1\right)\beta - n\alpha\right)}{n+1} - \frac{\cos\alpha - \cos\left(\left(n-1\right)\beta - n\alpha\right)}{n-1}\right] \tag{13}$$

$$b_n = \frac{V_m}{2\pi}\left[\cos\frac{n\pi}{3} - \cos\frac{2n\pi}{3} + 2\right]\left[\frac{\sin\alpha - \sin\left(\left(n+1\right)\beta - n\alpha\right)}{n+1} + \frac{\sin\alpha + \sin\left(\left(n+1\right)\beta - n\alpha\right)}{n-1}\right] \tag{14}$$

The RMS value of the nth harmonic component of phase-a voltage, V_{asn}, is obtained from:

$$V_{asn} = \sqrt{\frac{a_n^2 + b_n^2}{2}} \tag{15}$$

Thus, closed-form analytical expressions of V_{as1} and V_{asn} can be obtained by substituting the analytical expression of β, eqn. (9), into eqns. (10), (11), (13) and (14).

2.2 Mode (0/2) $\left(\beta + \frac{\pi}{3} < \alpha < \frac{5\pi}{6}\right)$

In this mode, either 2 or no thyristors, Fig. 1, are conducting, i.e. either two lines or no lines of the motor are connected to the supply. The waveforms in Fig. 4 represent the motor phase-a voltage and current in this mode at a certain speed and triggering angle [6-11].

Motor phase-a voltage, v_{as}, in this mode, can be expressed as follows, Fig. 4 [7, 10]:

$$\begin{aligned}
v_{as} &= \frac{v_{AB}}{2} = \frac{\sqrt{3}}{2}V_m \sin\left(\omega t + \frac{\pi}{6}\right) && \alpha \le \omega t \le \beta + \frac{2\pi}{3} \\
&= 0 && \beta + \frac{2\pi}{3} \le \omega t \le \alpha + \frac{\pi}{3} \\
&= \frac{v_{AC}}{2} = \frac{\sqrt{3}}{2}V_m \sin\left(\omega t - \frac{\pi}{6}\right) && \alpha + \frac{\pi}{3} \le \omega t \le \beta + \pi \\
&= 0 && \beta + \pi \le \omega t \le \alpha + \pi \\
&= \frac{v_{AB}}{2} = \frac{\sqrt{3}}{2}V_m \sin\left(\omega t + \frac{\pi}{6}\right) && \alpha + \pi \le \omega t \le \beta + \frac{5\pi}{3} \\
&= 0 && \beta + \frac{5\pi}{3} \le \omega t \le \alpha + \frac{4\pi}{3} \\
&= \frac{v_{AC}}{2} = \frac{\sqrt{3}}{2}V_m \sin\left(\omega t - \frac{\pi}{6}\right) && \alpha + \frac{4\pi}{3} \le \omega t \le \beta + 2\pi \\
&= 0 && \beta + 2\pi \le \omega t \le \alpha + 2\pi
\end{aligned} \tag{16}$$

2.2.1 Analytical Determination of the Extinction Angle in Mode (0/2)

Applying the same procedure followed in subsection (2.1.1) for mode (2/3), an analytical expression of the extinction angle, β, in this mode can be obtained. In this case the numerical results used to derive the appropriate analytical expression of β are obtained from the solution of the following transcendental equation in β [5-11]:

$$\sin\left(\beta - \phi - \frac{\pi}{6}\right) + \sin\left(\alpha - \phi + \frac{\pi}{6}\right)e^{-\left(\beta - \alpha + \frac{2\pi}{3}\right)\cot\phi} = 0 \tag{17}$$

For this mode of operation, the derived explicit analytical expression which fits the numerical obtained values of β as a function of α and ϕ is [9, 12]:

$$\beta = \frac{\pi}{6} - \frac{\left(\alpha_c - \frac{\pi}{2}\right)}{\left(\alpha_c - \phi\right)^3 - \left(\frac{5\pi}{6} - \phi\right)^3}\left(\left(\frac{5\pi}{6} - \phi\right)^3 - \left(\alpha - \phi\right)^3\right) \tag{18}$$

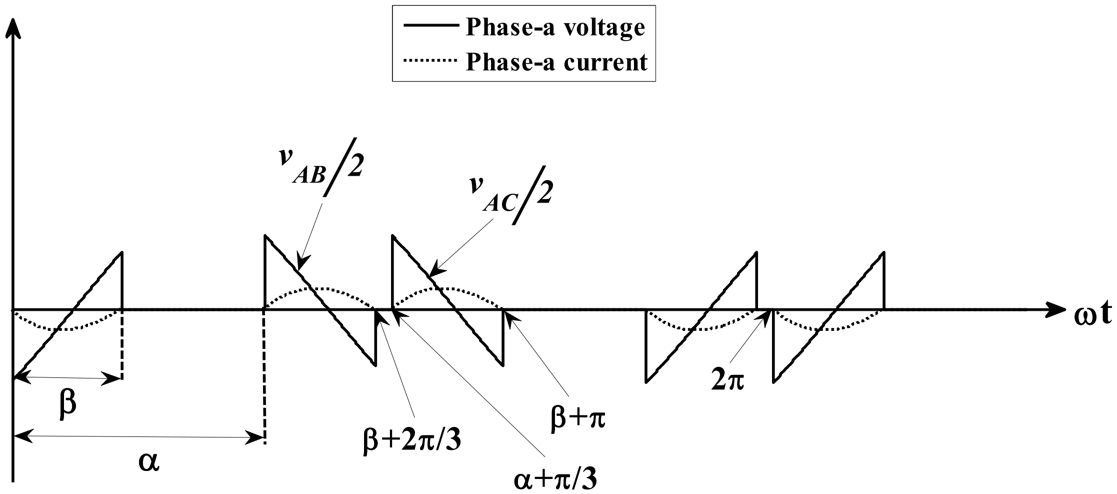

Fig. 4. Induction motor phase-a voltage and current waveforms, $\phi = 45^\circ$, $\alpha = 120^\circ$ (Mode 0/2)

2.2.2 Derivation of Analytical Expressions of the Voltage in Mode (0/2)

Referring to Fig. 4, The Fourier coefficients, a_1 and b_1, of the fundamental component of the motor phase-a voltage, in this mode, are as follows [11]:

$$a_1 = \frac{V_m}{\pi}\left[\left(\frac{3}{2}\beta - \frac{3}{2}\alpha + \pi\right)\sin\alpha + \left(\frac{3}{4}\sin(\beta-\alpha) - \frac{3\sqrt{3}}{4}\cos(\beta-\alpha)\right)\sin\beta\right] \tag{19}$$

$$b_1 = \frac{V_m}{\pi}\left[\left(\frac{3}{2}\beta - \frac{3}{2}\alpha + \pi\right)\cos\alpha + \left(-\frac{3}{4}\sin(\beta-\alpha) + \frac{3\sqrt{3}}{4}\cos(\beta-\alpha)\right)\cos\beta\right] \tag{20}$$

The RMS value of the fundamental component of the motor phase-a voltage, V_{as1}, can be obtained as follows:

$$V_{as1} = \sqrt{\frac{a_1^2 + b_1^2}{2}} \tag{21}$$

For the nth harmonic component [11]:

$$\begin{aligned} a_n = \frac{\sqrt{3}V_m}{2\pi}\Bigg\{ & \Bigg[\sin\left((n+1)\beta - n\alpha + (2n+1)\frac{\pi}{3}\right) \\ & + \sin\left((n+1)\beta - n\alpha + (3n+1)\frac{\pi}{3}\right) \\ & - \sin\left(\alpha - \frac{\pi}{3}\right) - \sin\left(\alpha + (n-1)\frac{\pi}{3}\right)\Bigg] / (n+1) \\ & + \Bigg[\sin\left((n-1)\beta - n\alpha + (2n-1)\frac{\pi}{3}\right) \\ & + \sin\left((n-1)\beta - n\alpha + (3n-1)\frac{\pi}{3}\right) \\ & - \sin\left(\frac{\pi}{3} - \alpha\right) - \sin\left((n+1)\frac{\pi}{3} - \alpha\right)\Bigg] / (n-1)\Bigg\} \end{aligned} \tag{22}$$

$$\begin{aligned} b_n = \frac{\sqrt{3}V_m}{2\pi}\Bigg\{ & \Bigg[\cos\left(\alpha - \frac{\pi}{3}\right) + \cos\left(\alpha + (n-1)\frac{\pi}{3}\right) \\ & - \cos\left((n+1)\beta - n\alpha + (3n+1)\frac{\pi}{3}\right)\Bigg] / (n+1) \\ & + \Bigg[\cos\left(\frac{\pi}{3} - \alpha\right) + \cos\left((n+1)\frac{\pi}{3} - \alpha\right) \\ & - \cos\left((n-1)\beta - n\alpha + (2n-1)\frac{\pi}{3}\right) \\ & - \cos\left((n-1)\beta - n\alpha + (3n-1)\frac{\pi}{3}\right)\Bigg] / (n-1)\Bigg\} \end{aligned} \tag{23}$$

The RMS value of the nth harmonic component of phase-a voltage is obtained from:

$$V_{asn} = \sqrt{\frac{a_n^2 + b_n^2}{2}} \quad (24)$$

Also, closed form analytical expression of V_{as1} and V_{asn} in this mode can be obtained by substituting the expression of β , eqn. (18), into eqns. (19), (20), (22) and (23).

2.3 Critical Triggering Angle

The triggering angle at which the operation is transferred from the first mode of operation, mode (2/3), to the second mode, mode (0/2), is known as the critical triggering angle, α_c , and it can be obtained from [5-8, 10, 11]:

$$\alpha_c = \beta + \frac{\pi}{3} \quad (25)$$

Substituting eqn. (25) into eqn. (5), the critical triggering angle can be obtained from the following equation [10]:

$$\alpha_c = \phi + \tan^{-1} \frac{\sqrt{3}}{1 + 2K_6 e^{\frac{\pi}{3}\cot\phi}} \quad (26)$$

On the other hand, if eqn. (25) is substituted into eqn. (17), the critical triggering angle can, also, be obtained as follows [10]:

$$\alpha_c = \phi + \tan^{-1} \frac{2e^{\frac{\pi}{3}\cot\phi} - 1}{\sqrt{3}} \quad (27)$$

2.4 Electromagnetic Torque of the Thyristor-Controlled Induction Motor

After obtaining the fundamental component of the phase-a voltage, whether the operating mode is mode (2/3) or mode (0/2), the fundamental component of the stator phase current, I_{as1} , can be obtained from the following equation [1, 13]:

$$I_{as1} = \frac{V_{as1}}{Z_{in}} \quad (28)$$

where Z_{in} is the induction-motor equivalent impedance, and can be obtained from the following equation:

$$Z_{in} = \sqrt{R_{in}^2 + X_{in}^2} \quad (29)$$

The corresponding rotor phase current, I'_{ar1} , is obtained as follows, Fig. 2 [1, 13]:

$$I'_{ar1} = I_{as1} \cdot \frac{X_m}{\sqrt{\left(\frac{R'_r}{s}\right)^2 + \left(X_m + X'_{lr}\right)^2}} \quad (30)$$

The electromagnetic torque due to the fundamental component of the motor phase voltage is obtained from [1, 13]:

$$T_{e1} = \frac{3I'^2_{ar1}\frac{R'_r}{s}}{\omega_{ms}} \quad (31)$$

where ω_{ms} is the synchronous angular speed corresponding to the fundamental frequency in mech. rad/s.

For the fifth-harmonic component of the motor phase voltage, whose phase sequence is reversed, the slip can be obtained as follows [1]:

$$s_5 = \frac{6 - s}{5} \quad (32)$$

The per-phase equivalent circuit of the three-phase induction motor corresponding to the fifth-harmonic component of its phase voltage is shown in Fig. 5:

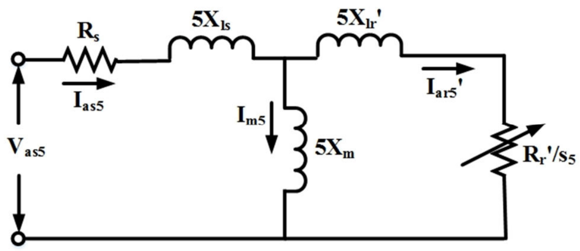

Fig. 5. Steady-state per-phase equivalent circuit of the three-phase induction motor corresponding to the fifth-harmonic component of its phase voltage.

The fifth-harmonic component of the stator phase current, I_{as5} , is obtained from the following equation:

$$I_{as5} = \frac{V_{as5}}{Z_{in5}} \quad (33)$$

where Z_{in5} is the per-phase input impedance of the motor corresponding to the fifth-harmonic component of its phase voltage, Fig. 5.

The corresponding rotor phase current, I'_{ar5} ,is obtained using the following expression, Fig. 5:

$$I'_{ar5} = I_{as5} \cdot \frac{5X_m}{\sqrt{\left(\frac{R'_r}{s_5}\right)^2 + \left(5X_m + 5X'_{lr}\right)^2}} \tag{34}$$

The electromagnetic torque due to the fifth-harmonic component of the motor voltage is obtained from:

$$T_{e5} = \frac{3I'^2_{ar5}\frac{R'_r}{s_5}}{5\omega_{ms}} \tag{35}$$

The seventh-harmonic component of the motor phase voltage has the same phase sequence as that of the fundamental component, so the seventh-harmonic component slip can be obtained from [1]:

$$s_7 = \frac{6+s}{7} \tag{36}$$

The per-phase equivalent circuit of the motor corresponding to the seventh-harmonic component of its phase voltage is shown in Fig. 6:

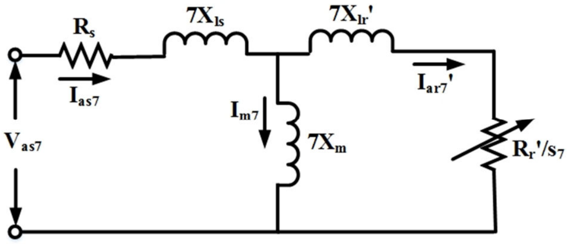

Fig. 6. Steady-state per-phase equivalent circuit of the three-phase induction motor corresponding to the seventh-harmonic component of its phase voltage.

The seventh-harmonic component of the stator phase current, I_{as7}, can be obtained from:

$$I_{as7} = \frac{V_{as7}}{Z_{in7}} \tag{37}$$

where Z_{in7} is the per-phase input impedance of the motor corresponding to the seventh-harmonic component of its phase voltage, Fig. 6.

The corresponding rotor phase current, I'_{ar7}, is obtained as follows, Fig. 6:

$$I'_{ar7} = I_{as7} \cdot \frac{7X_m}{\sqrt{\left(\frac{R'_r}{s_7}\right)^2 + \left(7X_m + 7X'_{lr}\right)^2}} \tag{38}$$

The electromagnetic torque due to the seventh-harmonic component of the motor voltage is obtained from:

$$T_{e7} = \frac{3I'^2_{ar7}\frac{R'_r}{s_7}}{7\omega_{ms}} \tag{39}$$

The total electromagnetic torque of the three-phase induction motor, T_e, if the fifth-and seventh-harmonic torques are the only harmonic torques taken into consideration, is obtained from [13]:

$$T_e = T_{e1} - T_{e5} + T_{e7} \tag{40}$$

3. Results

MATLAB programming language is used to obtain the performance characteristics of a phase-controlled induction motor (M1), whose parameters are given in Appendix A.

To check the validity of the analytical approach presented to determine the extinction angle, β, the numerical solution of eqn. (5), mode (2/3), and of eqn. (17), mode (0/2), for a range of α was obtained for motor speeds of 1307, 1069 and 1496 rpm which correspond to phase angles of 45˚, 60˚ and 75˚ respectively. The results obtained were compared with previously published results obtained numerically [5] for an R-L load with ϕ =45˚, ϕ =60˚and ϕ =75˚. The two sets of results are found to be almost identical as shown in Fig. 7 (a).

Then, closed-form analytical expressions in eqns. (9) and (18) corresponding to mode (2/3) and mode (0/2), respectively, were used to obtain the extinction angle, and the computed results were compared with the numerical results as shown in Fig. 7 (b). It is evident from this figure that the two sets of results are almost identical.

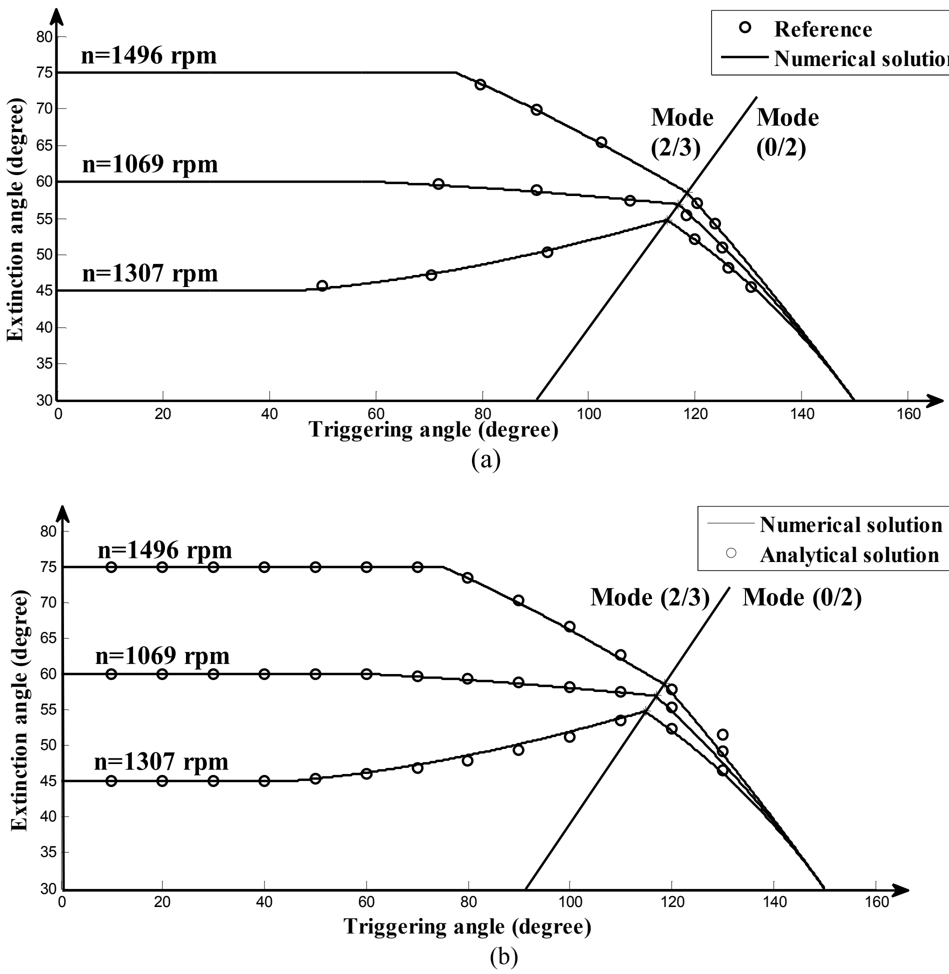

Fig. 7. Extinction angle-triggering angle characteristics for various motor speeds.

The performance characteristics of the induction motor under consideration are obtained at speeds 600, 1000, 1200 and 1300 rpm. Table 1 shows these speeds and the corresponding phase angles, at fundamental frequency, obtained using eqn. (3).

Speed (rpm)	Phase angle (degree)
600	69.24˚
1000	62.266˚
1200	53.654˚
1300	45.733˚

Table 1. Various speeds of the induction motor M1 and the corresponding phase angles at fundamental frequency.

The relationship between the extinction angle and the triggering angle for various values of the motor speed is shown in Fig. 8. The results for mode (2/3) and for mode (0/2) are obtained numerically using eqns. (5) and (17) respectively, and are obtained analytically using eqns. (9) and (18) respectively.

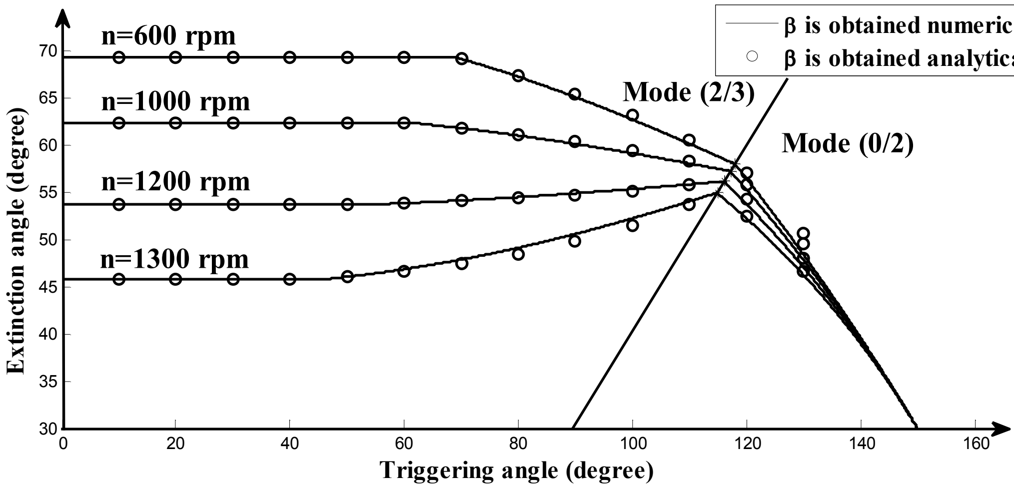

Fig. 8. Extinction angle-triggering angle characteristics for various phase angles.

It can be noticed from Fig. 8 that the numerical and analytical results are almost identical. It can also be noticed that there are two modes of operation, mode (2/3) and mode (0/2), where the operation is transferred from the first mode to the second one at the critical triggering angles 117.98°, 117.21°, 116.12° and 114.9° which are corresponding to the speeds 600, 1000, 1200 and 1300 rpm respectively.

Using the values of the extinction angle obtained numerically and analytically for several motor speeds and several triggering angles, the relationship between the fundamental component of the motor phase voltage and the triggering angle for various values of the motor speed are obtained either using eqns. (10), (11) and (12) for mode (2/3) or eqns. (19), (20) and (21) for mode (0/2). The results are shown in Fig. 9.

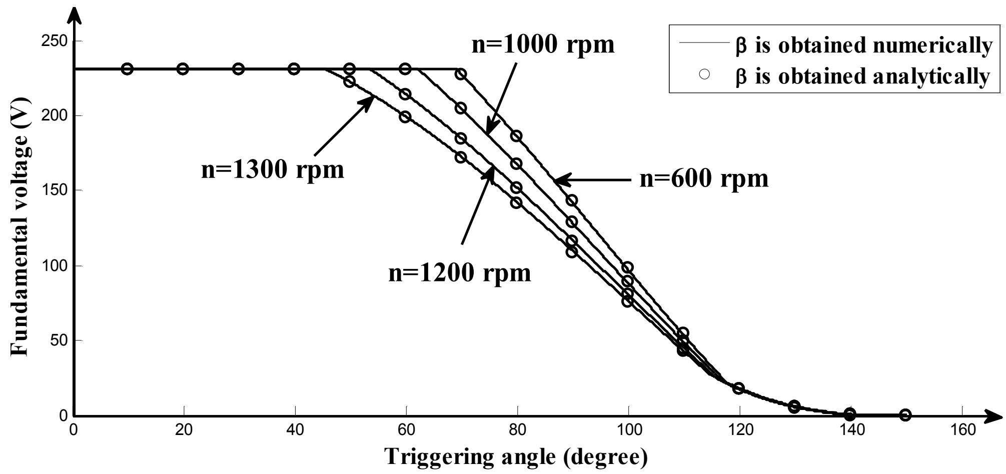

Fig. 9. Fundamental component of phase voltage-triggering angle characteristics for various motor speeds.

It is evident from Fig. 9 that the two sets of results are almost identical.
The fundamental stator current-triggering angle characteristics for various motor speeds are obtained using eqn. (28) where the fundamental voltage, in this equation, is obtained using the numerical and analytical values of the extinction angle. The results are shown in Fig. 10.

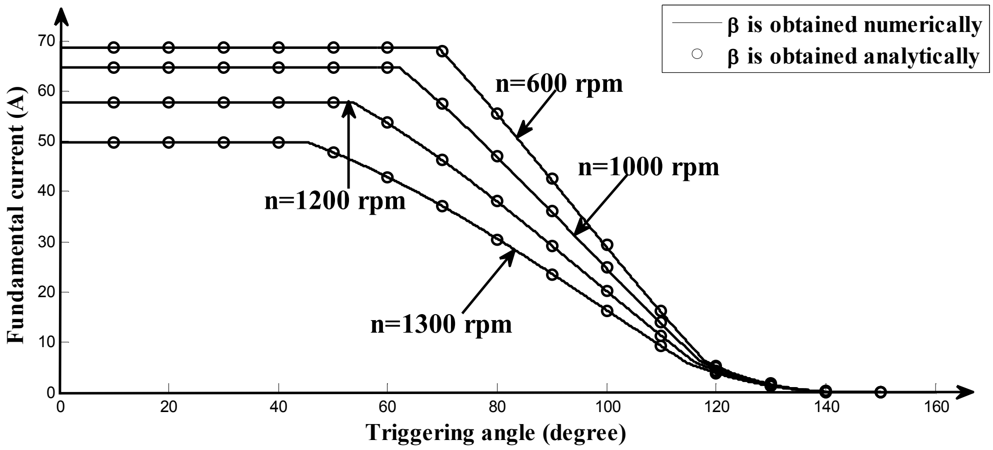

Fig. 10. Fundamental stator current-triggering angle characteristics for various motor speeds.

It can be noticed from Fig. 10 that the two sets of results are almost identical.

The developed torque corresponding to the fundamental component of the motor voltage-triggering angle characteristics for various motor speeds are obtained using eqn. (31) where the rotor current, in this equation, is computed using the obtained values of the stator current, Fig. 10, for both the numerical and analytical values of the extinction angle. The results are shown in Fig. 11.

It can be noticed from Fig. 11 that there is almost no difference between the two sets of results of the electromagnetic torque, corresponding to fundamental components, using both the numerical and analytical values of the extinction angle.

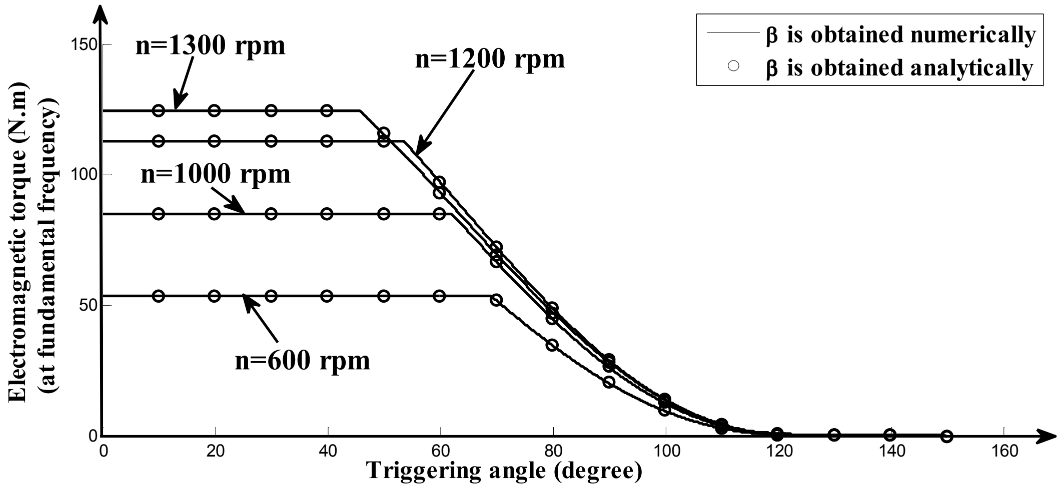

Fig. 11 Electromagnetic torque (at fundamental frequency)-triggering angle characteristics for various motor speeds

The total electromagnetic torque of the induction motor, if the fifth-and seventh-harmonic torques are the only considered harmonic torques, is obtained using eqn. (40). As the fifth-and seventh- harmonic torques can be neglected with respect to the developed torque corresponding to the fundamental component of the motor voltage as noticed from the computed values shown in Table 2, the total electromagnetic torque of the induction motor can be assumed to be equal only to the fundamental torque without losing accuracy in its computation.

Motor Speed (rpm)	Phase angle (ϕ)	Triggering angle (α)	Fundamental Torque T_1 (Nm)	Fifth-harmonic torque T_5 (Nm)	Seventh-harmonic torque T_7 (Nm)
600	69.24°	25°	53.33	0	0
		50°	53.33	0	0
		75°	43.06	0.055	0.008
		100°	9.475	0.02	0.007
1000	62.266°	25°	84.77	0	0
		50°	84.77	0	0
		75°	55.46	0.0435	0.0077
		100°	12.48	0.012	0.0075
1200	53.654°	25°	112.74	0	0
		50°	112.74	0	0
		75°	60.4	0.031	0.0057
		100°	13.94	0.02	0.0072
1300	45.733°	25°	124.61	0	0
		50°	115.76	0.033	0.0051
		75°	48.4	0.021	0.0038
		100°	13.78	0.00043	0.0065

Table 2. Fundamental and harmonic torques for various triggering angles at various phase angles of the induction motor M1.

4. Conclusion

In this paper, closed-form analytical expressions for the extinction angle of a phase-controlled induction motor, using thyristors, in terms of its phase angle and the triggering angle of the switching elements in various modes of operation are presented.

The results obtained from these expressions were compared with those obtained numerically, and the two sets of results were in close agreement.

The fundamental voltage, current and the corresponding torque-triggering angle characteristics for various motor speeds are obtained using the extinction angle values obtained numerically and those obtained analytically, and the two sets of results were compared, and were found to be in very close agreement.

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Appendix A

Induction Motor (M1) [14]

7500-W, 400-V, 50-Hz, 16-A, 4-pole, 3-phase induction motor whose parameters are:

$R_s = 0.6\ \Omega$ $\quad$ $R'_r = 0.4\ \Omega$

$X_m = 37.7\ \Omega$ $\quad$ $X_{ls} = 0.9425\ \Omega$

$X'_{lr} = 2.325\ \Omega$ $\quad$ $J = 0.05\ kg.m^2$